Introduction to Beam Echo

http:\\www.slac.stanford.edu\~stupakov\talks.html

Gennady Stupakov

SLAC

ICFA – HB2002

April 8-12, 2001

Outline of the talk

- Introduction
- “Shadow” echo
- Longitudinal echo of a coasting beam
- Transverse echo, emittance recovering
- Longitudinal echo of a bunched beam
- Conclusion

Introduction

Echo can be observed in a medium without dissipation which exhibits oscillations or waves. Usually these oscillations decohere due to the frequency spread in the ensemble of the oscillators. Excite the frequency ω_1, wait until the oscillations decohere, excite the frequency ω_2. The echo signal at the frequency $n\omega_2 \pm m\omega_1$ can be observed long after the second signal decoheres.

Echo effect in various media

- Spin echo in solids

  E. L. Hahn, Phys. Rev. 80, 580 (1950)

- Photon echo in solids and gases

  N. A. Kurnit, I. D. Abella and S. R. Hartmann, Phys. Rev. Lett. 13, 567, (1964)

- Plasma wave echo

  Hill R. M. and Kaplan D. E. Phys. Rev. Lett. 14, 1061, (1965)

- Echo in a liquid with gas bubbles

  I. A. Kotel'nikov (1985)

- Echo in accelerators

  G.V. Stupakov, SSCL-579 (1992)

  G.V. Stupakov and K. Kauffmann, SSCL-587 (1992) http://www.desy.de/~hoff/hoff/ECHOS/

Papers on Beam Echoes

References

- Measurement Setup for Bunched Beam Echoes in the HERA Proton Storage Ring
  E. Vogel, W. Kriens, and U. Hurdelbrink, Report DESY-HERA-00-09 (2000) and in Report DESY-HERA-00-07 (2000)
- Transverse Beam Echo Measurements on a Single Proton Bunch at the SPS
  G. Arduini, F. Ruggiero, F. Zimmermann, and M. P. Zorzano, SL-Note-2000-048-MD (2000)
- Possible Quantum Mechanical Effect on Beam Echo
  A. Chao and B. Nash, SLAC-PUB-8726 (2000)
- Transverse Echoes in RHIC
  W. Fischer, B. Parker, and O. Bruening, Proceecdings of US-LHC Collaboration Meeting: Accelerator Physics Experiments for Future Hadron Colliders (2000)
- Echo
  G. V. Stupakov, in Handbook of Accelerator Physics and Engineering, A. W. Chao and M. Tigner (eds.) (1999)
- Measurements of Intrabeam Scattering Rates below Transition in the Fermilab Antiproton Accumulator
  C. Bhat, L.K. Spenzouris and P.L. Colestock, Proceedings of PAC99, New York (1999) Proceedings of PAC99, New York (1999)
- Beam Echo Measurements
  L. K. Spentzouris, P. L. Colstock, and C. Bhat, Proceedings of PAC99, New York (1999)
- Bunched Beam Echoes in the AGS
  J. Kewisch and M. Brennan, Proceedings of EPAC98, Stockholm (1998)
- Effect of Diffusion on Bunched Beam Echo
  G. V. Stupakov and A. W. Chao, Proceedings PAC97, Vancouver (1997)
- Beam Echoes in the CERN SPS
  O. Bruening, T. Linnecar, R. Ruggiero, W. Scandale, El Shaposhnikova, D. Stellfeld, Proceedings PAC97, Vancouver and Report CERN-SL-97-023-AP (1997)
- Measuring diffusion coefficients and distribution functions using a longitudinal beam echo
  O. Brüning, T. Linnecar, F. Ruggiero, W. Scandale and E. Shaposhnikova, presented at the Workshop on Nonlinear and Collective Phenomena in Beam Physics, Arcidosso, 1996
- Longitudinal beam echo in the CERN SPS
  O. Brüning, T. Linnecar, F. Ruggiero, W. Scandale, E. Shaposhnikova and D. Stellfeld, CERN-SL-96-51 AP (1996) and Proc. EPAC Conference, Sitges (Barcelona), 1996, eds. S. Myers, A. Pacheco, R. Pascual, Ch. Petit-Jean-Genaz, and J. Poole (IOP, Bristol, 1996), pp. 1332-1334.
- Direct Measurement of Diffusion Rates in High Energy Synchrotrons Using Longitudinal Beam Echoes
  L. K. Spentzouris, J.-F. Ostiguy, and P. L. Colstock, PRL 76 (4) pp. 620--623 (1996)
- Longitudinal beam echo measurements in a coasting beam in the SPS
  O. Brüning, T. Linnecar, F. Ruggiero, W. Scandale, E. Shaposhnikova and D. Stellfeld, CERN SL-MD note 217 (1996).
- Measurement of longitudinal beam echoes in a coasting beam in the SPS
  O. Brüning, T. Linnecar, F. Ruggiero, W. Scandale, E. Shaposhnikova and D. Stellfeld, CERN SL-MD note 206 (1996).

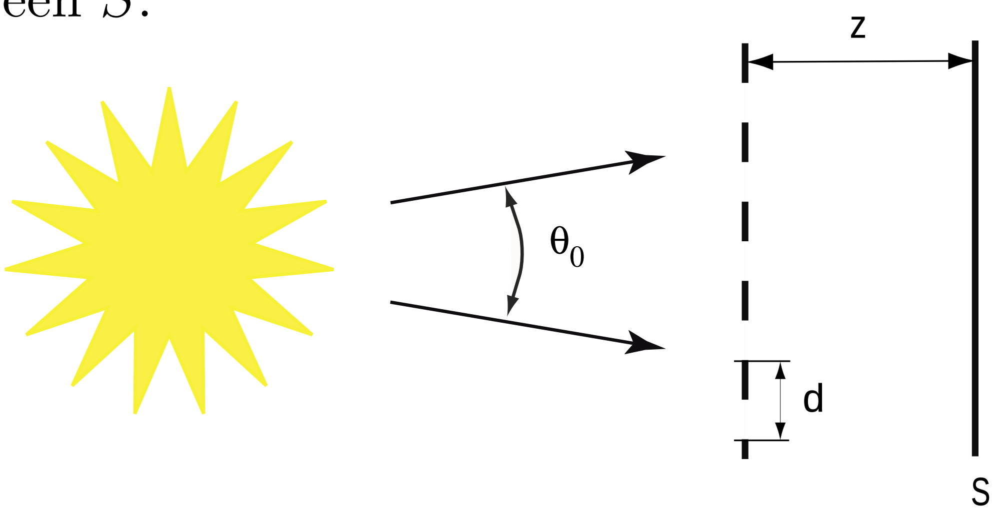
- Longitudinal echo in a continuous beam (extension to more general case)
  E. Shaposhnikova, CERN SL/note 96-03 (RF) (1996)
- On the longitudinal echo in a continuous beam
  E. Shaposhnikova, CERN SL/note 95-125 (RF) (1995)
- A proposal to measure longitudinal beam echoes in the SPS
  O. S. Bruening, F. Ruggiero and W. Scandale, CERN SL/note 95-115 (AP) (1995)
- On the possibility of measuring longitudinal echoes in the SPS
  O. S. Bruening, Report CERN-SL-95-83 (1995)
- Nonlinear Wave Phenomena in Coasting Beams
  P.L. Colestock, L.K. Spentzouris and F. Ostiguy, PAC 95 (1995)
- Echo Effect in Accelerators
  G. V. Stupakov and K. Kauffmann, Report SSCL-Preprint-238 (1993)
- Echo Effect in Accelerators
  G. V. Stupakov and K. Kauffmann, Report SSC-587 (1992)
- Echo Effect in Hadron Colliders
  G. V. Stupakov, SSCL-579 (1992)

Links

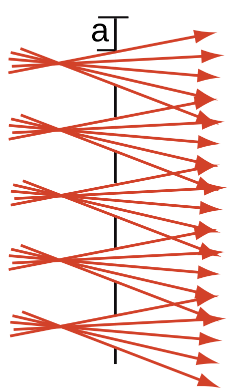
- Beam Echo Measurements at CERN

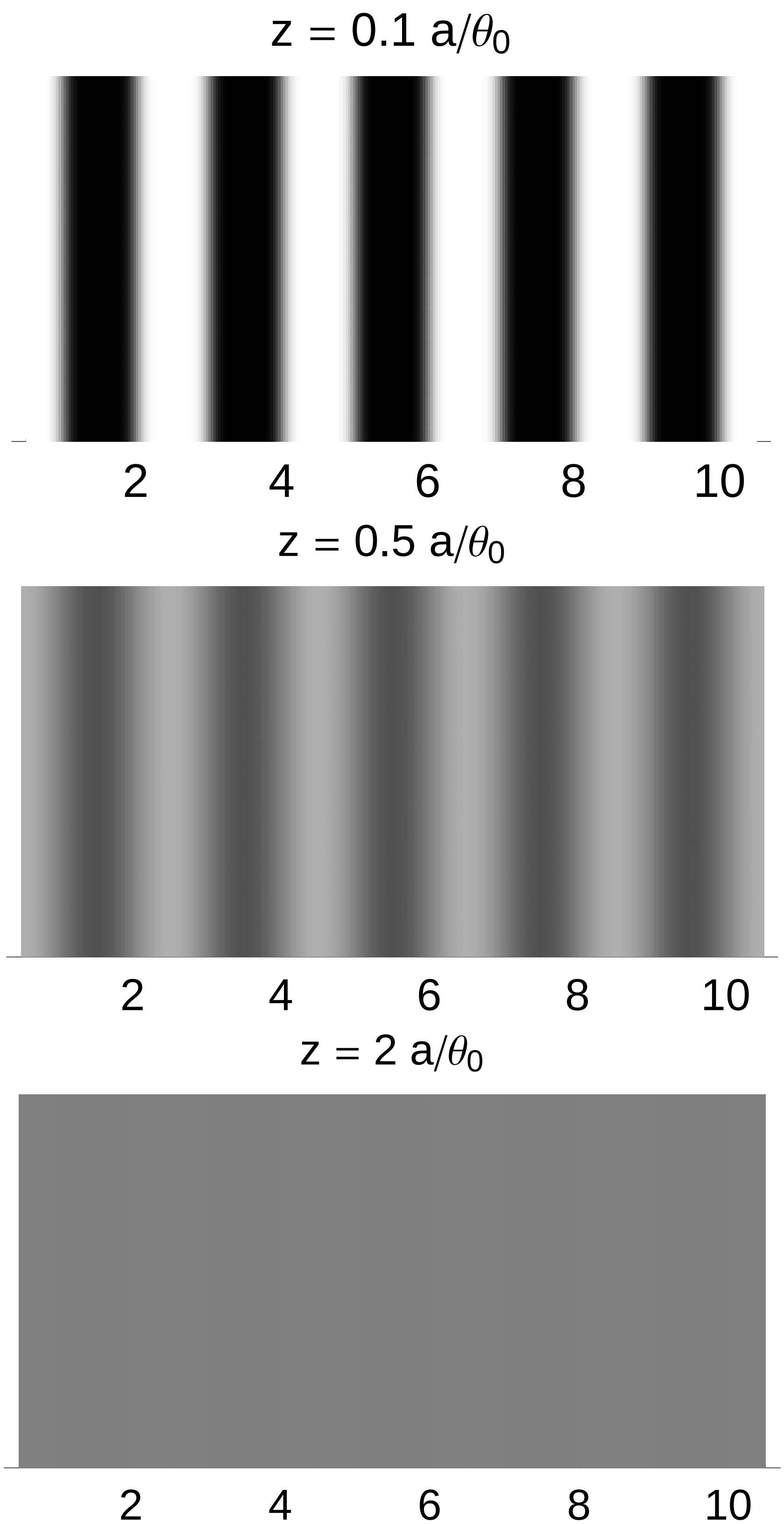
“Shadow” echo

A thin mask with periodic slits of period d is uniformly illuminated by light with angular spread θ_0. Neglect diffraction and interference ($\lambda \ll d$). The image is observed at the screen S.

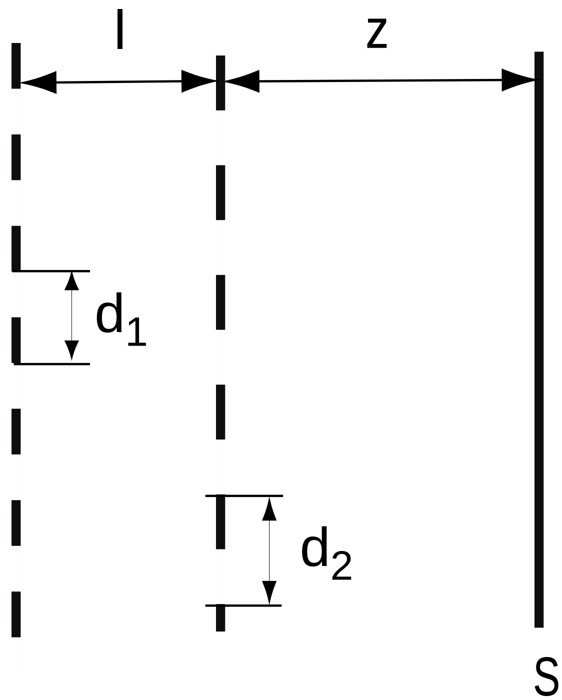


If $z > a/\theta_0$, the shadow areas smear out.




z = 0.1 a/θ0
2
4
6
8
10
z = 0.5 a/θ0
2
4
6
8
10
z = 2 a/θ0
2
4
6
8
10

Add another screen with period d_1 at distance l from the first one ($l, z \gg a/\theta_0$).



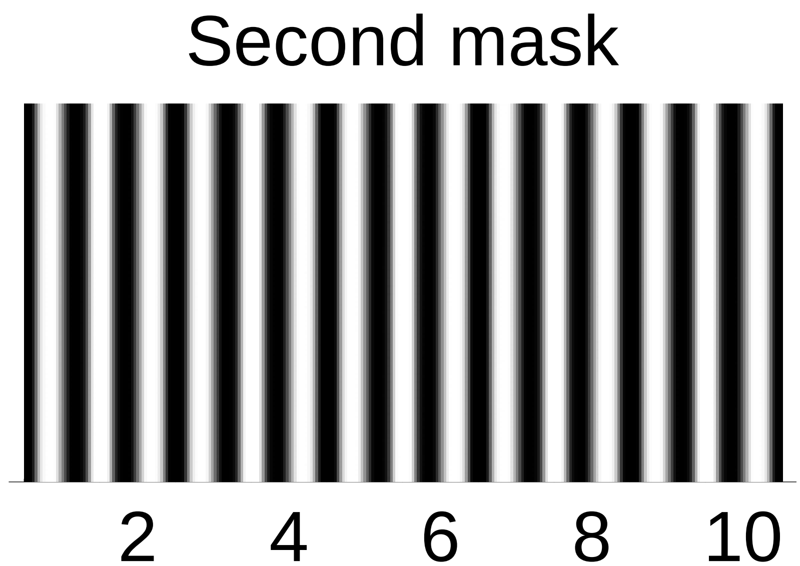
If the screen is at the right position, one can observe a pattern on the screen — *shadow echo*:

$$z = \frac{\frac{n}{d_1}}{\frac{m}{d_2} - \frac{n}{d_1}} l,$$

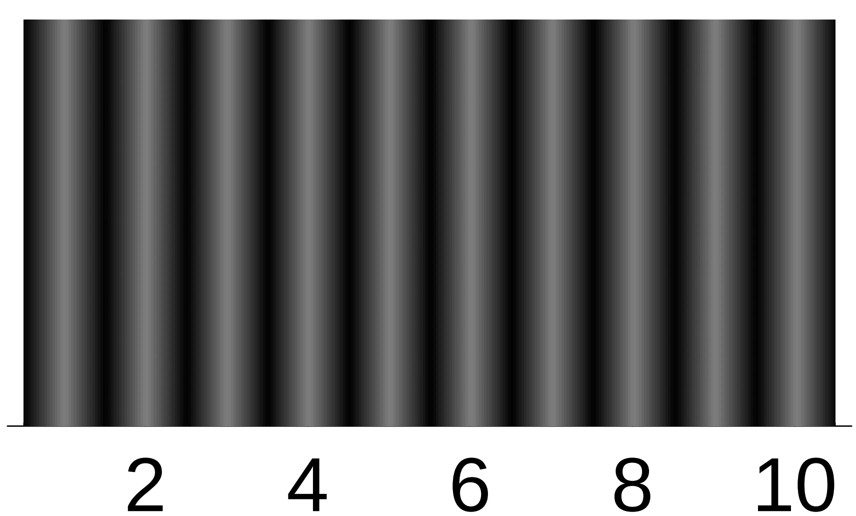
where m and n are integers. The patten has a period d

$$\frac{1}{d} = \left| \frac{n}{d_1} - \frac{m}{d_2} \right|.$$

Example: $d_2 = \frac{1}{3} d_1$.

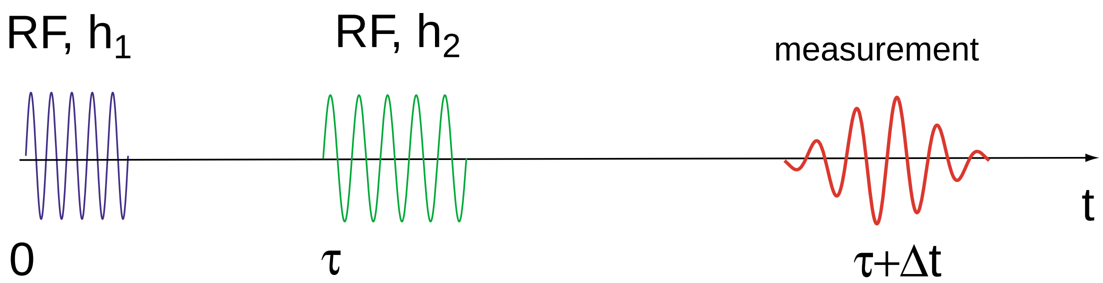


For $m, n = 1$, echo should be observed at $z = \frac{1}{2} l$ with the period $d = \frac{1}{2} d_1$



Longitudinal Echo for a Coasting Beam

Experiments at Fermilab, CERN.



Apply RF kick at frequency $\omega_1 = h_1\omega_0$

Wait time $\tau \gg \tau_{d1}$

Apply RF kick at frequency $\omega_2 = h_2\omega_0$

Wait time $\Delta t \gg \tau_{d2}$

Measure beam modulation

Echo theory

θ – angular position in the ring
$p = \frac{\Delta E}{E}$
$f_0(p)$ – equilibrium distribution function
σ_p – characteristic width of $f_0(p)$ (rms)

1. First RF kick

$$p \to p + \Delta p_1 \sin(h_1 \theta)$$

where $\Delta p_1 = eV_{RF}\omega_1 \Delta t_{RF}/2\pi E \ll \sigma_p$

2. Wait time τ

$$\theta \to \theta + p\frac{\tau}{T}$$

where T is the "mixing time"

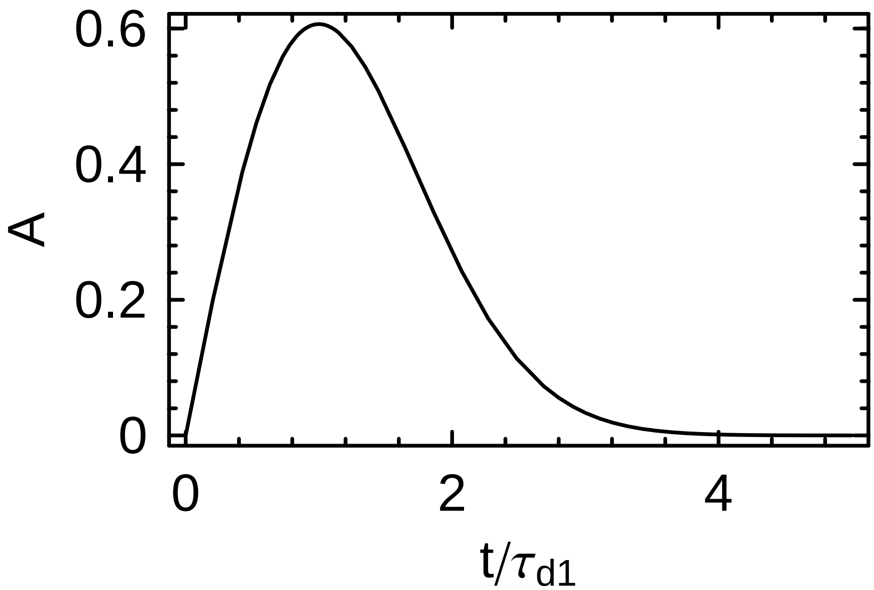
$$\frac{1}{T} = \frac{\omega_0 \eta}{\beta^2} \qquad \eta = \frac{1}{\gamma_t^2} - \frac{1}{\gamma^2}$$

If the beam modulation is measured after time t, then the signal is

$$\frac{\Delta I}{I} = -\frac{\Delta p_1}{\sigma_p} A\left(\frac{\tau}{\tau_{d1}}\right)\cos h_1\theta$$

where

$$\tau_{d1} = \frac{T}{h_1 \sigma_p}$$



3. Second RF kick

$$p \rightarrow p + \Delta p_2 \sin(h_2\theta)$$

This signal decoheres after time

$$\tau_{d2} = \frac{T}{h_2\sigma_p}$$

4. Echo can be observed around time

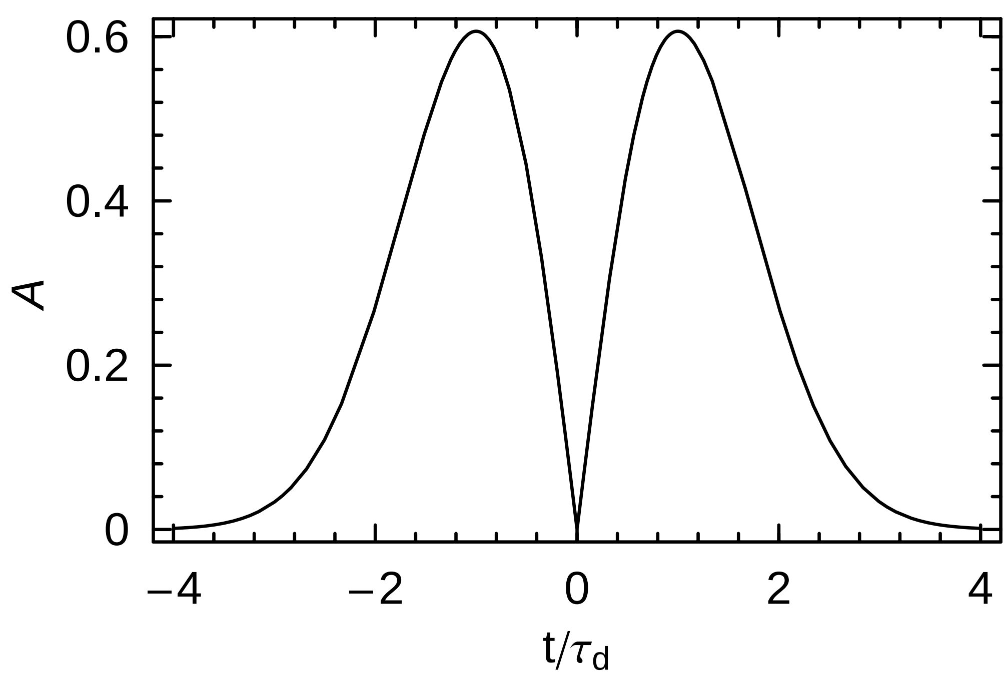
$$\Delta t_{\text{echo}} = \frac{h_1\tau}{nh_2 - h_1}$$

where n is an integer. We assume $n = 1$.

Echo signal

$$\frac{\Delta I}{I} = -\frac{\Delta p_1}{\sigma_p} J_1(x) A \left(\frac{t - t_{\mathrm{echo}}}{\tau_d} \right) \cos(h_2 - h_1)\theta$$

$$x = h_1 \Delta p_2 \frac{\tau}{T} \qquad \tau_d = \frac{T}{\sigma_p (h_2 - h_1)}$$

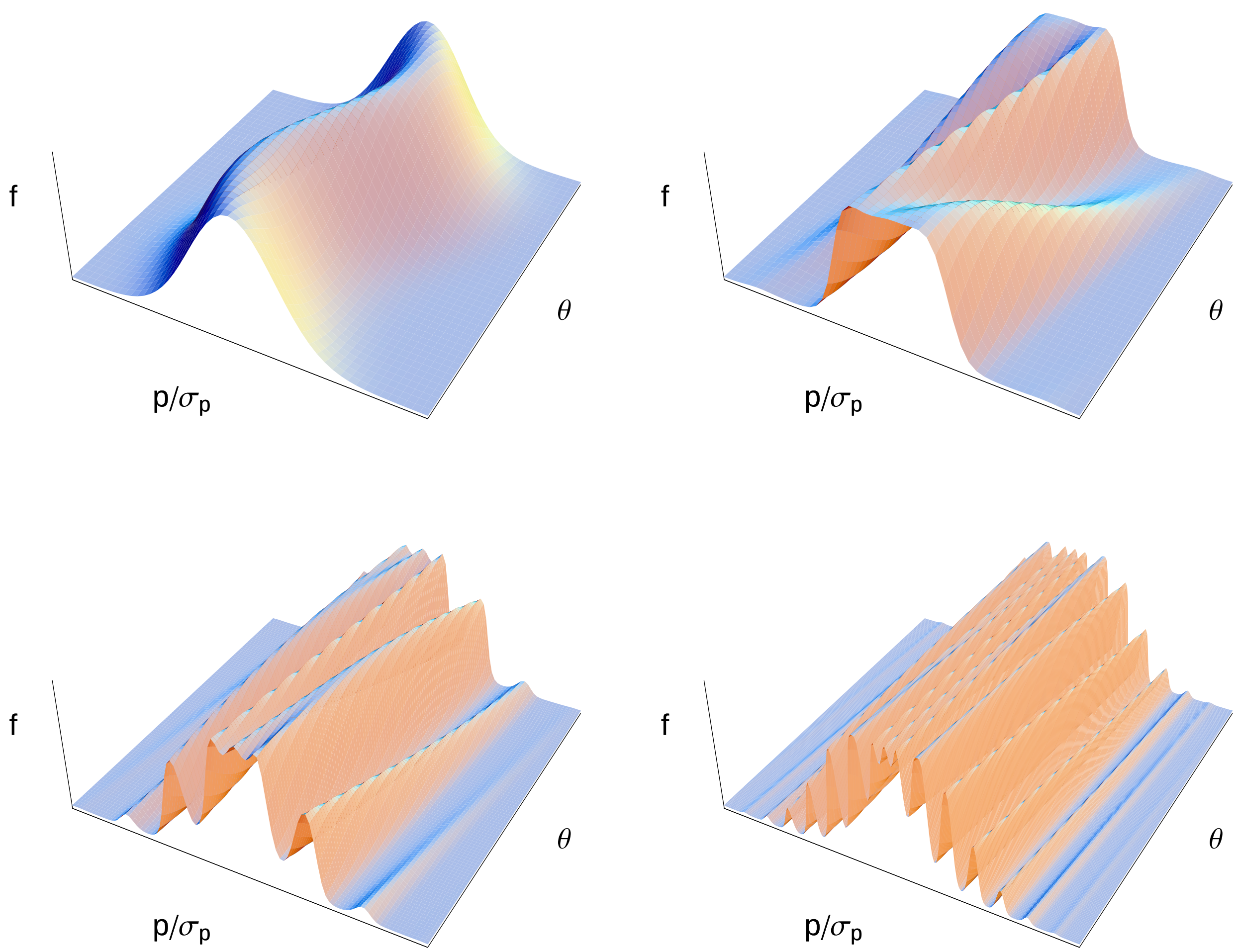


Remarkably, even if $\Delta p_2 \ll \Delta p_1$, one recover $\max(J_1(x)) \approx 0.6$ of the original signal ($x = 1.8$). Maximum of the echo signal: $\tau \propto 1/\Delta p_2$.

Effect of Diffusion on Echo

Echo is very sensitive to energy diffusion of particles in the beam.

Beam distribution function $f(\theta, p, t)$ after an RF kick.



Modulation in p at the time τ of the second kick

$$\Delta p_{\mathrm{mod}} \sim \frac{T}{h_1 \tau}$$

If there is diffusion, with diff. coeff. $\mathcal{D}$, after time Δt_{echo} it smears out perturbations with

$$\Delta p_{\mathrm{diff}} \sim \sqrt{\mathcal{D} t_{\mathrm{echo}}}$$

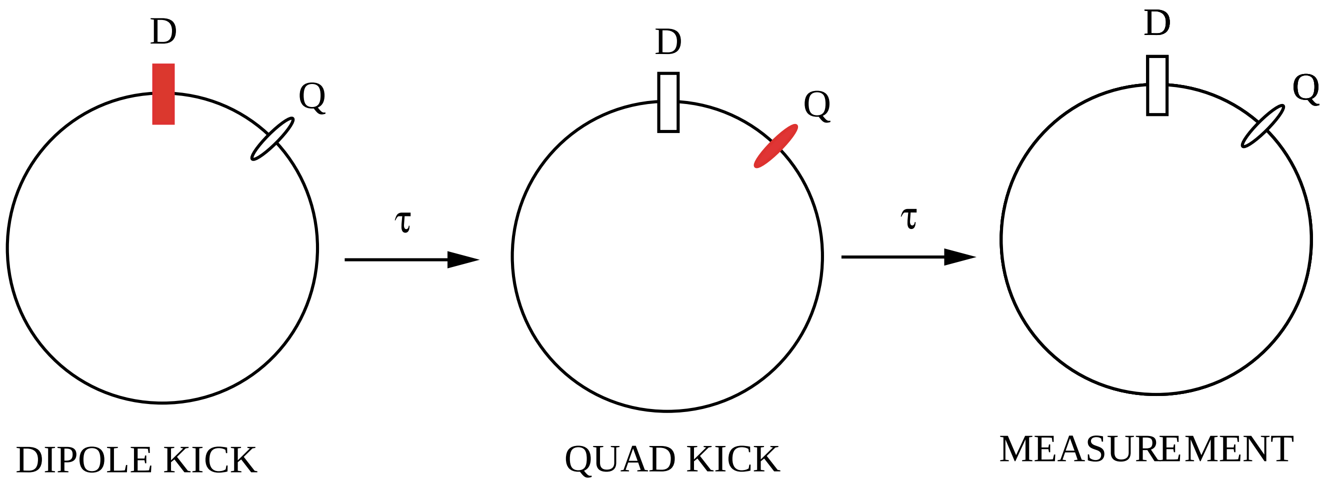
Diffusion destroys echo if $\Delta p_{\mathrm{diff}} > \Delta p_{\mathrm{mod}}$. The theory gives the echo signal suppression factor

$$\propto \exp\left(-\frac{1}{3}\frac{\Delta p_{\mathrm{diff}}^2}{\Delta p_{\mathrm{mod}}^2}\right) = \exp\left(-\frac{\mathcal{D} t_{\mathrm{echo}}^3}{3T^2}\frac{h_1^2}{h_2^2(h_2 - h_1)^2}\right)$$

$\mathcal{D} \sim 10^{-13}\ \mathrm{s}^{-1}$ were measured at CERN SPS — corresponds to longitudinal emittance doubling ~ 100 days!

Transverse Echo, Bunched Beam

Betatron oscillations



$\Delta t_{\text{echo}} = \tau$

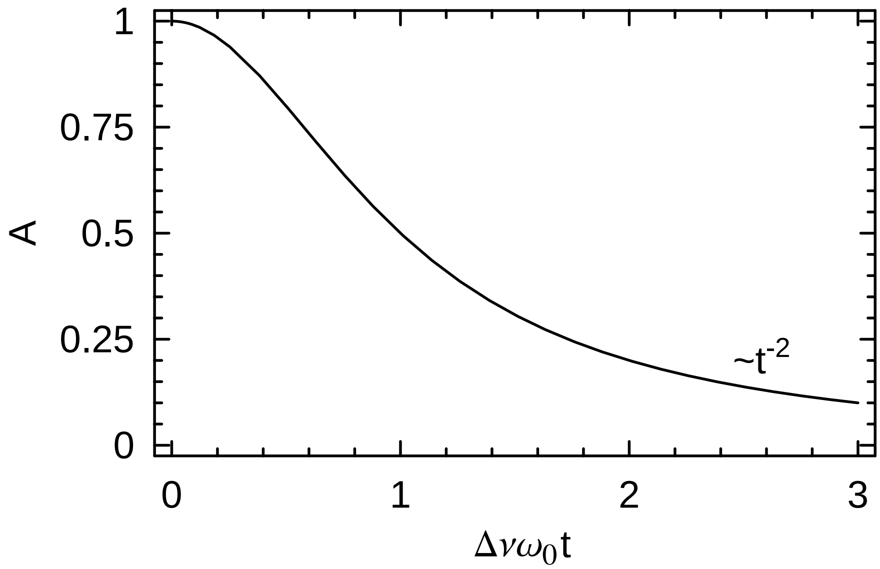
Perturbation theory of the transverse echo:

- Small dipole and quadrupole kicks
- Tune spread due to nonlinearity: $\nu = \nu_0 + \Delta\nu \frac{J}{J_0}$
- No diffusion

Transverse echo is sensitive to diffusion in the phase space.

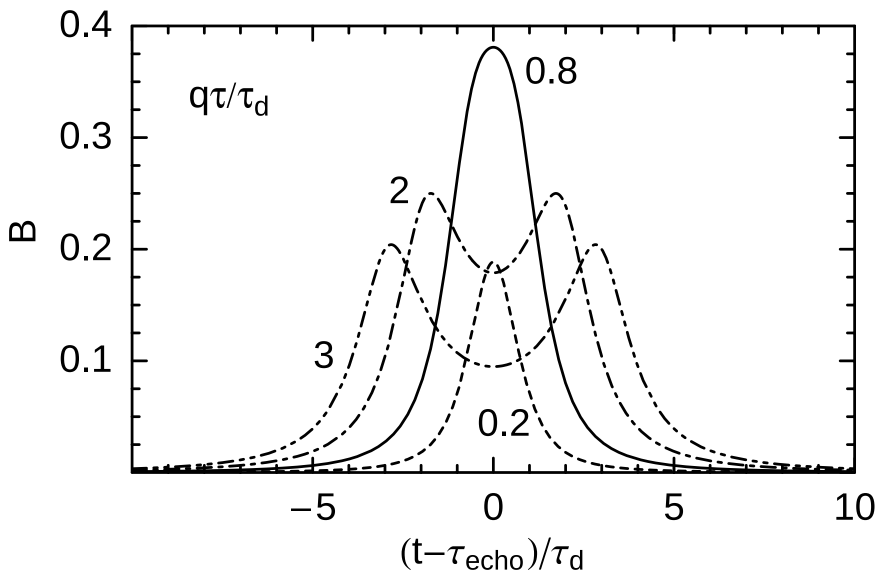
Dipole kick:

$$\bar{x} = A(t) \sin(\nu_0 \omega_0 t + \phi(t))$$

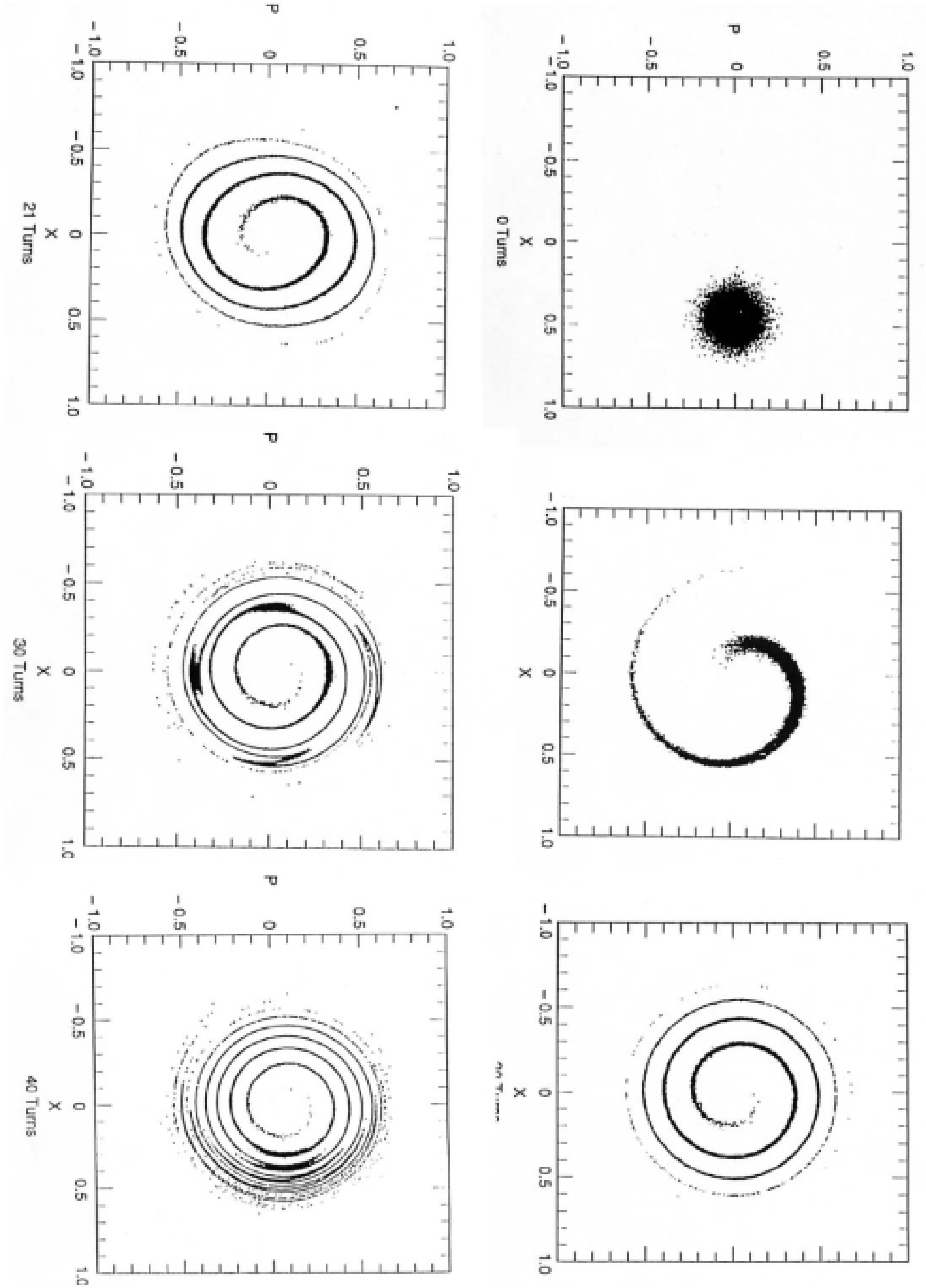


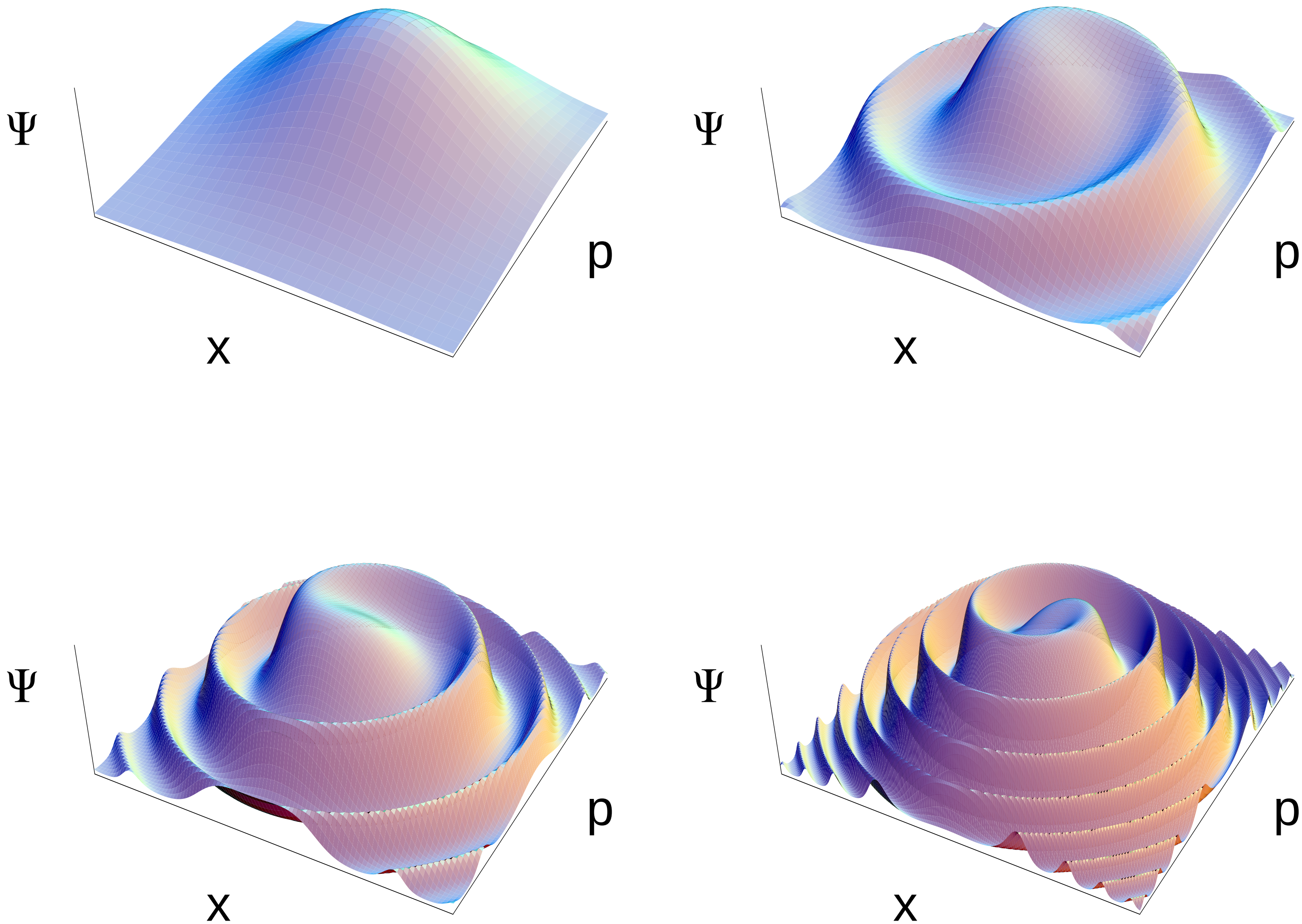
Decoherence time $\tau_d = (\omega_0 \Delta\nu)^{-1}$. Quad kick of strength $q = \beta/F$, F - focal length

$$\bar{x}_{\mathrm{echo}} = B(t - \tau_{\mathrm{echo}}, q) \sin(\nu_0 \omega_0 (t - \tau_{\mathrm{echo}}) + \phi(t))$$



One can have a strong echo with a weak quadrupole kick, $\tau \sim 1/q$.


P
X
0 Turns
21 Turns
30 Turns
40 Turns


Ψ
p
x
Ψ
p
x
Ψ
p
x
Ψ
p
x

Recovering Beam Emittance with Echo

Assume that the beam is injected in the ring with an offset a. After decoherence time τ_d the offset translates into increased beam emittance

$$\Delta\epsilon = \epsilon_0 \frac{a^2}{2\sigma_0^2}$$

To avoid emittance dilution one needs a feedback that takes the offset out at $t \ll \tau_d$.

Question: can we recover some of the emittance increase at much later time using echo? How much?

The answer:

$$q\tau = 0.4\tau_d, \qquad t = t_{\text{echo}}$$

The echo signal is $0.33a$.

$$\Delta\epsilon = (1 - 0.26)\epsilon_0 \frac{a^2}{2\sigma_0^2}.$$

Longitudinal Echo of a Bunched Beam

$x \to z$

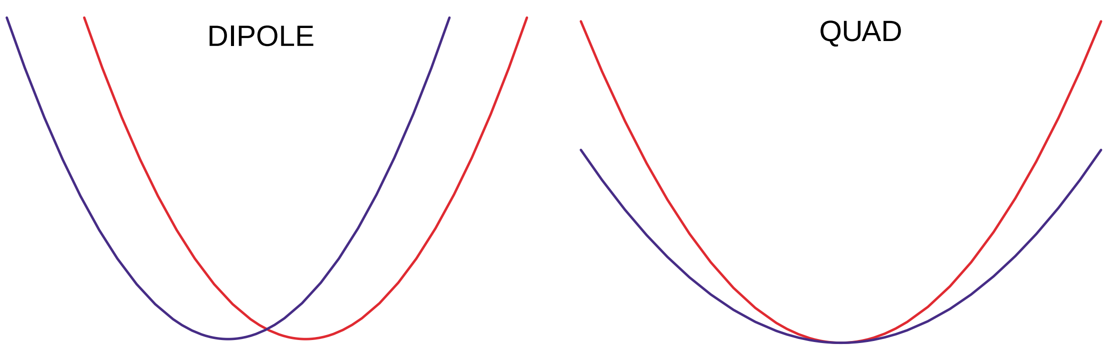
$x' \to \delta$

$\omega_\beta \to \omega_s$

Nonlinearity $\omega_\beta(J) \to \omega_s(J)$

Dipole kick $\to$ change of RF phase

Quad kick $\to$ change of RF voltage



Conclusion

- Beam echo is a member of the echo family, that "reverses time" and recoheres part of the original signal much later after the oscillation disappear. The echo can be observed in longitudinal and transverse directions, and in both bunched an coasting beams.
- The echo is sensitive to diffusion/dissipation in the beam and can be used as a diagnostic tool of extremely small diffusion.
- Other applications of echo are possible, for example, partial emittance recovering due to injection error on the time scale much larger than the decoherence time.